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Collision Risk Posed to the Global Positioning System by Disposal Orbit Instability

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Recent studies show that disposal orbits used by the global positioning system can be unstable, resulting in penetration of the operational constellation by the disposal orbit perigees. The purpose of this study was to obtain a preliminary understanding of the associated collision risk between disposed and operating vehicles. Collision risk was determined via direct statistical simulation, which involves determination of conjunction miss distance distributions. Study results include time histories of constellation penetration by disposal orbit perigees and an estimate of collision risk growth over 180 years.

Nomenclature

A_{CC}	=	average collision cross section between primary and threat objects
a	=	semimajor axis
d	=	miss distance
e	=	eccentricity
i	=	inclination
$N(d, T)$	=	cumulative number of miss distances over time interval T at a miss distance $\leq d$
p_C	=	collision probability from kinetic theory
p_T	=	total average collision probability with constellation
T	=	time interval
t	=	time
u	=	argument of latitude, $\omega + \nu$
v	=	average relative velocity between primary and threat objects
β	=	probability distribution fit parameter
ν	=	true anomaly
ρ	=	threat object spatial number density
ω	=	argument of perigee

Introduction

NEAR the end of their functional lives, satellites in the global positioning system (GPS) are placed in disposal orbits above the operational constellation. The purpose of this end-of-life (EOL) disposal procedure is to preclude the possibility of collision between the decommissioned and operating satellites. In addition to decommissioned satellites, spent upper stages of evolved expendable launch vehicles (EELVs) may also be placed into disposal orbits after they serve their mission of inserting future GPS Block IIF replacement vehicles into drift orbits near the targeted mission orbits.

Studies performed by Gick and Chao[1] and Chao[2] indicate that the GPS disposal orbits can be unstable and undergo significant eccentricity growth over several decades. The amount of eccentricity growth depends on the particular disposal orbit. In some cases the disposal orbit perigee can penetrate into the shell of the operational constellation, thereby producing a collision risk between the disposed and operating vehicles. Gick and Chao[1] recommend disposal orbit targeting strategies to restrict the disposal orbit eccentricity growth with the goal of preventing or delaying penetration of the GPS operational shell.

The eccentricity growth is strongly dependent on the initial eccentricity, argument of perigee, and right ascension of ascending node (RAAN = Ω). Figure 1 shows the maximum eccentricity achieved over 200 years as a function of RAAN for a disposal orbit that has an initial perigee 500 km above the ideal GPS operational orbit and an initial eccentricity of 0.005. (Throughout this paper perigee altitudes will be given in terms of height above the ideal GPS operational circular orbit with a radius of 26559.7 km.) In this sample case the disposal orbit insertion epoch was 1 August 2001, and the argument of perigee was selected to maximize the eccentricity growth. From this plot it is seen that, for the sample initial epoch and eccentricity, the GPS/EELV disposal orbit planes A, B, C, and F can attain (over time) eccentricities that are roughly an order of magnitude larger than those of planes D and E during the same time period. Maximum eccentricity can be viewed as a measure of disposal orbital stability. In extreme cases, where the eccentricity becomes particularly large (as much as 0.5 in 140 years), the disposal orbits can come close to both low-Earth-orbit (LEO) and geosynchronous-orbit operational altitudes. This large eccentricity growth is attributed to a resonance effect between sun–moon perturbations and nodal and apsidal regression caused by the secular component of the J_2 Earth gravitational harmonic.

The purpose of this study is to obtain a preliminary understanding of the risk of collision that would be posed to the operational constellation if disposed vehicles were inserted into the 500-km-perigee disposal orbit. This disposal orbit is the lower bound of one of two medium-Earth-orbit disposal regions recommended by the current U.S. Government Debris Mitigation Guidelines. The other region has an upper bound that is 500 km below the GPS ideal operational circular orbit. GPS operational requirements specify disposal orbit perigee altitudes higher than 500 km. Historically, some GPS vehicles have had adequate remaining propellant to raise perigee to as much as 1200 km or more at EOL.

Disposal Orbit Penetration of the Constellation

Collision between a disposed vehicle and an operational vehicle is possible only if their orbital altitude ranges overlap. For disposal orbits above the operational constellation, altitude overlap will occur if the perigee of the disposed vehicle falls below the apogee of the operational vehicle. Figure 2 shows a representative scatter plot of the apogee and perigee altitudes relative to the ideal GPS altitude of a sample Block IIF constellation. This plot was generated from recent orbit data on the 28 operational vehicles tracked by the GPS mission control segment (MCS). The apogees and perigees were computed from mean element data that were derived from osculating MCS state vectors dated 2000-8-14. (The osculating-to-mean conversion was performed by E. T. Campbell, a colleague of the authors.) The mean eccentricities were multiplied by a scale factor of 0.0275/0.02 because the EOL maximum eccentricity specification

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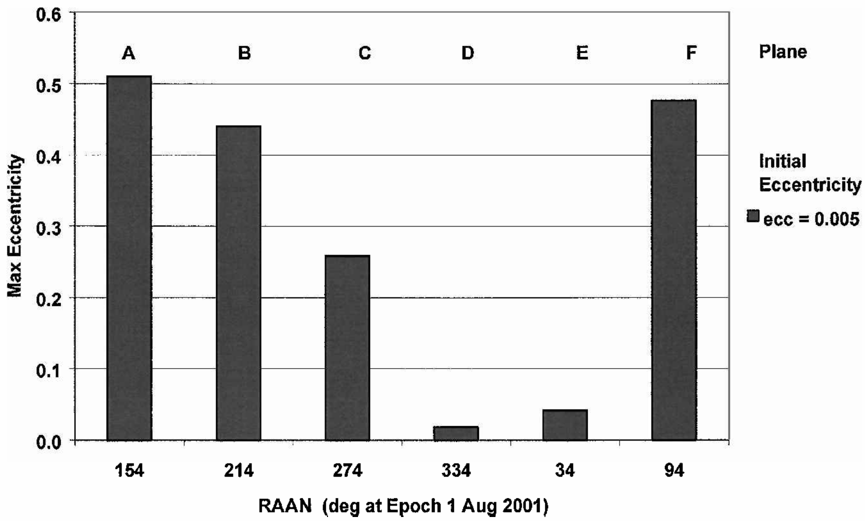

Fig. 1 Maximum eccentricity growth (worst case) over 200 years as a function of RAAN (initial perigee = 500 km above ideal GPS circular orbit, initial eccentricity = 0.005). GPS constellation planes (A–F) are shown for each value of RAAN at epoch.

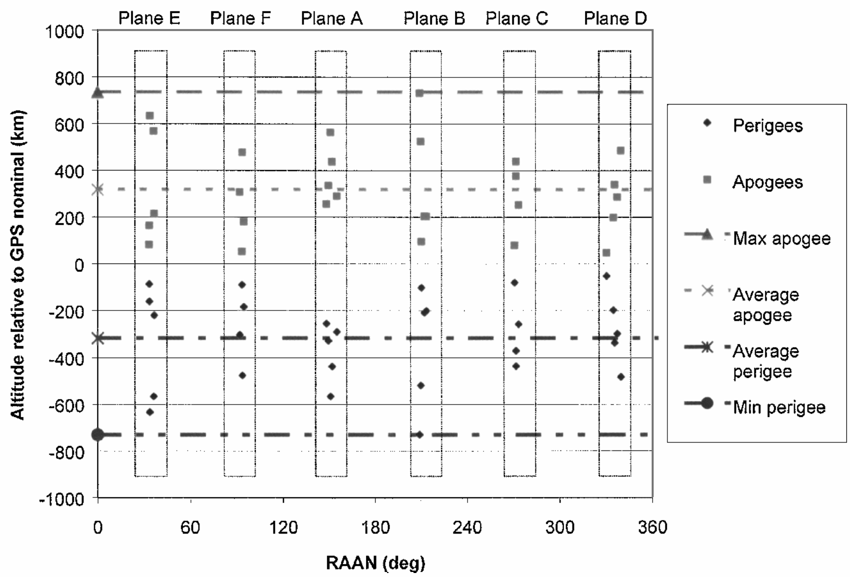

Fig. 2 Altitude spread of a sample representation of the GPS Block IIF constellation.

for GPS IIF vehicles was increased to 0.0275, whereas it is 0.02 for pre-Block IIF vehicles (Block II, IIA, and IIR). The resulting inflated eccentricities were limited to be no larger than 0.0275. The corresponding maximum and average apogees and perigees are shown by dotted lines in Fig. 2. The apogee and perigee values have a range of approximately ±730 km relative to the nominal altitude. This altitude bound corresponds to the maximum eccentricity requirement of 0.0275 for Block IIF vehicles.

Figure 3 shows the corresponding histogram of apogees and perigees using altitude bins of 50 km. Although there is some bunching near the average apogees and perigees, the distribution is relatively even through the maximum and minimum values and is closer to being uniform than, for example, Gaussian. The histogram is not completely symmetric about the ideal GPS altitude because each vehicle has some small deviation in its semimajor axis from the ideal 12-h orbit value because of stationkeeping residual error.

Figure 4 illustrates the penetration of the operational constellation over time by vehicles disposed at four different altitudes. Perigee histories of vehicles disposed on an epoch of 1 August 2001 in plane C ($\Omega = 274$ deg) at initial perigees of 500, 632, 832, and 1200 km are plotted. From Fig. 1 it is seen that this combination of plane and epoch has average eccentricity growth. In each case the initial disposal orbit eccentricity was selected to be 0.005. The eccentricity growth scenario was selected by determining the average eccentricity growth over a uniformly distributed random argument of perigee. The initial argument of perigee was selected to produce the average eccentricity growth scenario. The perigee histories were computed using the mean orbital element propagation software MEANPROP, which is a tool developed by The Aerospace Corporation that uses as its core module the Draper Semi-Analytic Orbit Propagator.[3] All pertinent perturbations were modeled: sun–moon gravity, solar radiation pressure, and an 8 × 8 WGS84 Earth gravity field.

The perigee altitude of 500 km corresponds to the recommendation of the U.S. Government Debris Mitigation Guidelines. The 632-km-perigee altitude is the GPS disposal orbit specification of pre-Block IIF satellites. The 832-km-perigee altitude would be a similar GPS disposal requirement for Block IIF. The 1200-km-perigee altitude represents recent decommissionings of GPS vehicles, which were boosted to higher altitudes to consume surplus propellant at EOL. The sample constellation apogee/perigee maximum and average deviations from the ideal GPS Block IIF orbit are shown as dotted lines. From this plot it is seen that the 500-km disposal orbit initial perigee lies within the GPS IIF operational constellation shell of ±730 km. It can also be seen that, even though the perigees start at different values, they all pass the lower boundary of the constellation during a relatively small time interval between 120 and 140 years. In other words, the perigees of the higher disposal orbits eventually overtake the perigees of the lower disposal orbits. Hence,

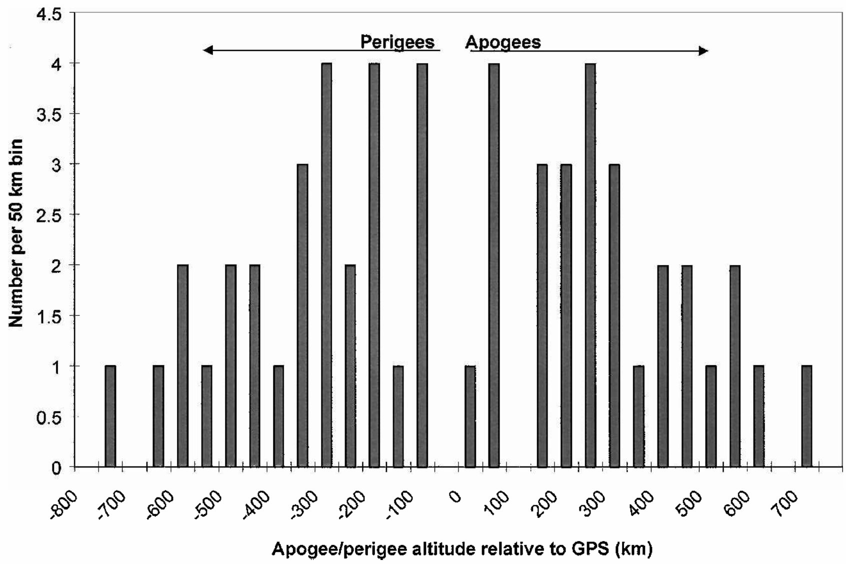

Fig. 3 Histogram of the apogees and perigees of a GPS Block IIF sample constellation.

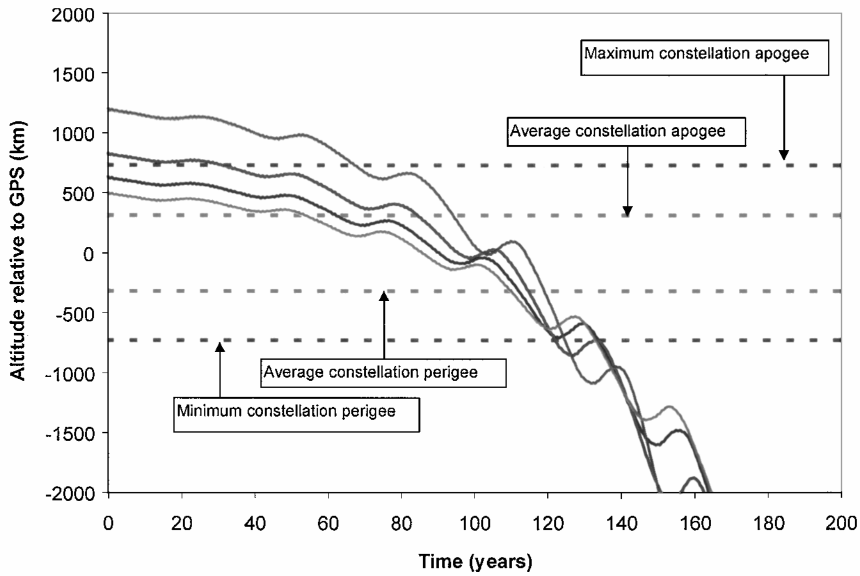

Fig. 4 Altitude penetration of a sample GPS Block IIF constellation by the perigees of four different candidate disposal orbits in plane C. (Initial argument of perigee was selected to yield average eccentricity growth. Initial disposal perigees are 500, 632, 832, and 1200 km.)

increasing the initial disposal perigee altitude has the effect of delaying the initial penetration of the constellation, but the subsequent penetration through the remainder of the constellation occurs more rapidly.

Collision Risk for Individual Disposed Vehicles

To assess the collision risk quantitatively, a statistical analysis of miss distance data was performed. The miss distance is the distance between the two objects when there is a local minimum in the separation distance as a function of time, that is, when a close approach or conjunction event occurs. A collision will occur if a conjunction has a miss distance that is small enough to permit the two objects to intersect each other physically. This depends not only on the miss distance between the centers of the two objects, but also on the geometrical layout and orientation of the two objects. The average keepout radius is computed to account for random orientation of both objects. In this study the keepout radius is averaged over the two collisional pairings of interest: operational GPS vs disposed GPS, and operational GPS vs disposed EELV upper stage. Hence, the probability of a close approach within a cited keepout radius is synonymous in this study with probability of collision.

In any probabilistic analysis it is necessary to identify separately uncertain or random aspects of the problem from those that can be treated deterministically. In this case the dominant source of uncertainty is the in-track position of the disposed vehicle relative to the operational vehicle. The initial argument of latitude u at disposal orbit insertion can vary widely depending on disposal epoch and orbit transfer strategy. Hence the initial argument of latitude is assumed in this study to be uniformly distributed over 360 deg. The initial conditions of the orbital parameters a, e, i, and Ω (semimajor axis, eccentricity, inclination, and RAAN, respectively) are treated deterministically. A value of argument of perigee ω is selected that yields the average eccentricity growth given deterministic values of the other orbital elements. The average eccentricity growth scenario was used to simplify the analysis of the effect of disposal orbit population growth on overall collision risk, as will be discussed later.

The dominant orbital perturbations affecting the long-term mean orbital evolution are the sun–moon perturbations, the gravitational harmonics of the Earth, and solar radiation pressure. Sun–moon and gravity harmonic perturbations can be modeled accurately, and hence the long-term mean orbital evolution is treated deterministically. The acceleration perturbation caused by solar radiation pressure cannot be predicted as accurately a priori because of the variability in attitude and surface solar reflectances of the disposed vehicle. However, MEANPROP simulation runs showed that, as a

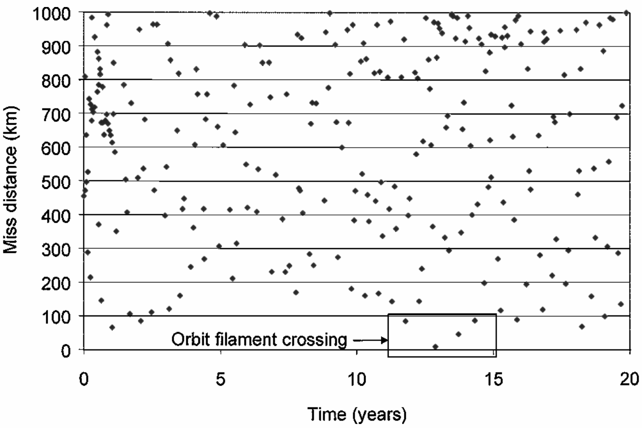

Fig. 5 Miss distance data over 20 years for operational vehicle SV13 vs a vehicle placed in a +500-km-perigee disposal orbit.

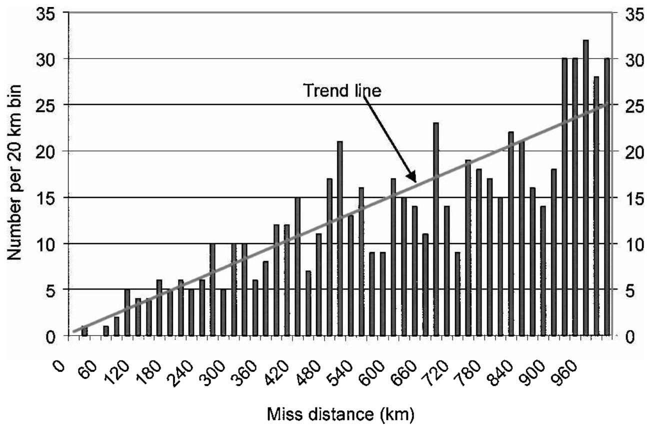

Fig. 6 Histogram of miss distance data over 20 years corresponding to data in Fig. 5.

result of the relatively small area-to-mass ratios of the vehicles involved, solar radiation pressure induced a maximum contribution to eccentricity of 0.02. This contribution is well below the eccentricity growth caused by sun–moon and Earth gravitational perturbations for most constellation planes (Fig. 1).

A simulation was used to compute and collect the miss distances between a disposed vehicle and a 28-member Block IIF sample operational constellation over a period of 180 years. The orbital evolution of the disposed vehicles was generated using MEANPROP. The orbits of the sample operational vehicles were modeled with a Keplerian/secular J2 propagator. This was done to represent constellation maintenance in a simple way by avoiding the long-term eccentricity growth and tesseral harmonic-induced station drift that would be included if MEANPROP were used. Therefore, the sample operational orbital elements were fixed except for RAAN and argument of perigee, which precess because of the secular J_2 effect.

As an example case, Fig. 5 shows the miss distance data over time for a vehicle disposed in the 500-km-perigee orbit in plane B ($\Omega = 214$ deg) with an initial eccentricity of 0.005 and a currently operational satellite SV13 (not a part of Block IIF). In this case the argument of perigee of the disposed satellite was selected to yield maximum eccentricity growth over 200 years. Each data point in the plot represents a close approach distance between the two satellites. The corresponding histogram of the miss distances is shown in Fig. 6 with an accompanying trend line.

The miss distance at a given conjunction is determined by two quantities: 1) the separation distance between the two orbit filaments and 2) the relative in-track phasing between the two vehicles. An orbit filament is defined as the orbit path that an object would sweep out if its five mean orbital elements a, e, i, Ω, and ω were frozen. The orbit filament separation distance is the minimum distance between any two points on the two filaments. As such, it serves as a lower bound on the miss distance between the satellites. The miss distance ensemble then cannot encompass distances below a given threshold until the orbit filament separation distance is below that threshold. The variation in the orbit filament separation distance is a function of the evolution of the mean orbital elements and hence is a deterministic quantity in this analysis.

The relative in-track phasing between two vehicles is primarily determined by the initial relative arguments of latitude at disposal orbit insertion and the difference in the mean motions of the two vehicles. A constant difference in mean motion results in repeated recirculation of the relative in-track phasing of the disposed vehicle with respect to the operational vehicle. For the 500-km-perigee disposal orbit with initial eccentricity of 0.005, a 360-deg recirculation occurs every 14.3 days. Hence 25.5 recirculations occur each year, that is, the relative in-track motion yields 25.5 opportunities for collision each year.

Figure 5 shows that there is a trend in the ensemble of miss distances. For example, no part of the ensemble falls below distances of 50 km over the 20-year period except for a brief interval between 12 and 14 years. This dip in the ensemble can be attributed to variation in the orbit filament separation distance. In the case of disposal orbits with growing eccentricity, the disposal orbit filaments will eventually cross the operational satellite filaments, thereby generating an opportunity for collision. Hence, in Fig. 5 the time point between 12 and 14 years where the ensemble almost touches the abscissa (a miss distance of zero) is very close in time to the orbit filament crossing.

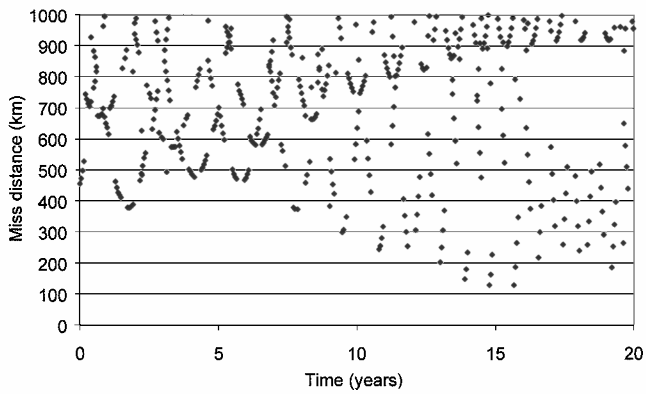

Fig. 7 Miss distance data over 20 years for operational vehicle SV15 vs a vehicle placed in a +500-km-perigee disposal orbit.

Some pairings of disposed and operational vehicles will have no orbit filament crossings during limited periods after disposal occurs. This is illustrated by the case shown in Fig. 7, which shows the miss distance data over time for a vehicle disposed in the 500-km-perigee orbit in plane B ($\Omega = 214$ deg) and a currently operational satellite SV15 (not a part of Block IIF). As in the preceding case, the initial eccentricity of the disposal orbit is 0.005, and the argument of perigee is selected to maximize eccentricity growth over 200 years. In this case the ensemble of miss distances never approaches the abscissa during the 20-year period. The data trend clearly shows that there is no possibility for collision during this period.

In most cases the relationship between the mean motions of the two vehicles will have a randomizing effect on the miss distances. In Fig. 8 the plotted points show the relative in-track differences between an operational vehicle and a vehicle disposed in the 500-km-perigee orbit at subsequent recirculation events. A recirculation event occurs whenever the disposed vehicle has completed a recirculation relative to the operational vehicle, and the operational vehicle is located at the point on its orbit corresponding to the orbit filament separation distance. For this case the angle difference is bounded by ± 6.212 deg. From the plot it is seen that angle difference points do not form horizontal lines, that is, they do not remain synchronized at fixed values. Also, across the four-year time period the points are closely spaced in the vertical direction. Hence, after four years the progression of points has already significantly swept out the space bounded by ± 6.212 deg. As a result, the range of possible miss distances corresponding

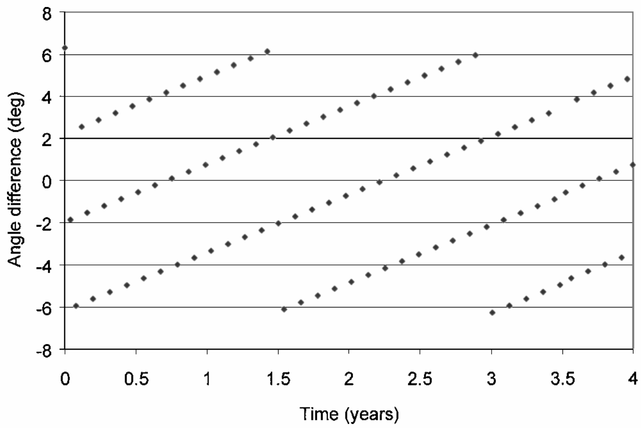

Fig. 8 Relative in-track angle difference between an operational vehicle and a vehicle placed in a +500-km-perigee disposal orbit at subsequent recirculations.

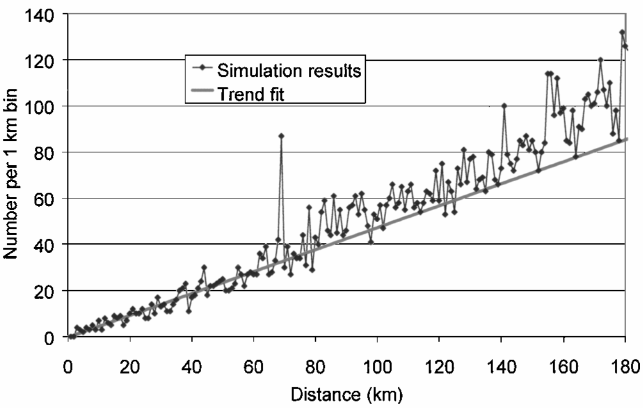

Fig. 9 Miss distance histogram over 180 years for all 28 Block IIF sample operational vehicles vs six vehicles (one per plane) placed in +500-km-perigee disposal orbits.

to the range of possible initial relative in-track positions will be well sampled over time (as many recirculations occur) by the miss distances that result for a single value of initial relative in-track position.

Because the range of miss distances will be well sampled over time for a single initial condition, it is possible to estimate a miss distance probability distribution from a single sample simulation accurately. Probability at collision level distances can then be computed from the estimated probability distribution. It is not necessary to repeat simulations for different values of initial relative argument of latitude as might be done in a Monte Carlo analysis. Avoiding a sweep of initial relative argument of latitude permits a tremendous reduction in the computational effort required to perform a probabilistic collision risk analysis over a period of decades. If the points in Fig. 8 formed horizontal lines and were not closely spaced in the vertical direction, the possible range of miss distances would not be well sampled over time, and it would be difficult to estimate the correct probability distribution from a single simulation. This scenario can occur for certain mean motion pairings, that is, for certain disposal orbit semimajor axes.

Figure 9 shows a histogram for miss distances between six disposed vehicles at 500 km (one per plane) and the entire 28 Block IIF sample operational constellation over a 180-year period. The arguments of perigee of the disposed vehicles were selected to yield average eccentricity growth. Because the histogram exhibits a clear linear trend, the cumulative histogram can be fit with a quadratic form with a single fit parameter $\beta(T)$:

$$N(d, T) = [d/\beta(T)]^2 \tag{1}$$

By modeling the miss distance points as the result of a large number of independent, random Bernoulli instances, and approximating the resulting collision probability formulation with a Poisson formulation, it can be shown that it is possible to use this equation to directly compute collision probability at small miss distances. Equations (18–23) of Ref. 4 illustrate an example of applying Bernoulli trials and a Poisson approximation in the formulation of collision probabilities.

This approach is very similar to that used by Chobotov and Johnson in a study of the collision risk between uncoordinated, collocated geosynchronous satellites.[5] In that study miss distance distributions were generated from propagations of orbital data and fit with a Weibull function. In that case use of the Weibull function to model the underlying probability distribution was indirectly justified by assuming that the asymptotic theory of extreme order statistics was applicable to the orbit data. In the approach used here, trend fitting of simulation results is directly justified by showing that the simulation results yield a good sampling of the underlying miss distance probability distribution.

Miss distance histograms were computed for 20-year intervals up to 180 years after disposal orbit insertion, and probability distribution fits were produced using a least-squares procedure. Table 1 contains the resulting variation in the fit parameter β as a function of time interval.

The average risk of collision p_T between one disposed satellite and all 28 members of the operational constellation over T years is computed from the probability distribution fits for six vehicles, one per plane, by dividing by six. This procedure effectively averages out the dependence on orbital plane ascending node:

$$p_T = 1 - \exp[-N(d, T)/6] \tag{2}$$

The variable d is then set equal to the keep-out radius.

Figure 10 shows the resulting average probability of collision over time between one disposed vehicle starting in a 500-km-perigee orbit and 28 operational constellation satellites. Average keep-out radii, assuming random orientation of the two vehicles at collision, were computed from the primary dimensions of the Block IIF and EELV upper stages. Collisions that generate significant amounts of debris were assumed to occur when the vehicle buses come

Table 1 Probability distribution fit parameter values

Time interval, years	Fit parameter β, km
0–20	9.159
20–40	5.567
40–60	4.496
60–80	3.405
80–100	2.692
100–120	2.425
120–140	2.164
140–160	2.091
160–180	2.051

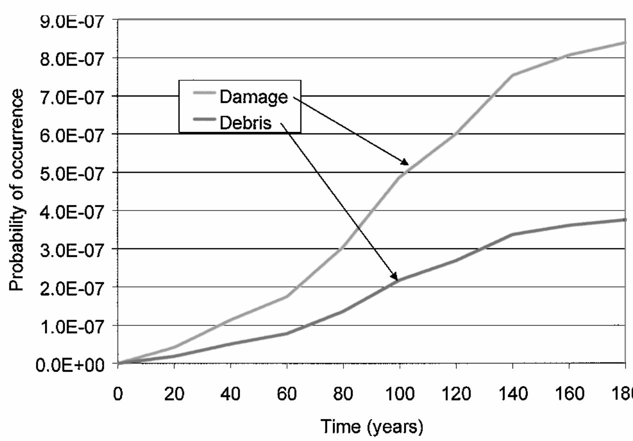

Fig. 10 Average probability of occurrence of damaging and debris-creating collisions over time between a single disposed vehicle and 28 operational GPS satellites (results from simulation and fitting process).

into contact. The corresponding average keep-out radius for debris-intensive collisions was estimated to be 3.1 m. Damaging collisions include debris-intensive collisions and the broader set of collisions, which generate small amounts of debris but significantly degrade or terminate satellite function. These would include collisions that cause solar array and antenna clipping. The average keep-out radius for damaging collisions was estimated to be 4.6 m.

In the simulation that generated the miss distances, the eccentricity growth of the operational vehicle orbits was not modeled, that is, the eccentricity was held constant at the value indicated by the MCS data prorated by the Block IIF maximum eccentricity specification. If the eccentricity of an operational vehicle were static, only a small number of filament crossings would be expected to occur over a 200-year period between its orbit and that of a given single disposed vehicle, primarily as a result of the slow relative nodal regression rate. In reality, however, the eccentricities of the operational vehicle orbits grow over time. In addition, every time an operational vehicle is replaced the eccentricity of the orbit is reset from its EOL value (worst case 0.0275) to its beginning-of-life value (typically 0.008).

This periodic growth and rectification of orbit eccentricity will induce additional orbit filament crossings. Although a larger number of filament crossings acts to increase the collision probability, this increase is generally negated by the accelerated relative rate of occurrence of each crossing. Later studies showed that including the effect of constellation eccentricity growth and rectification in the generation of miss distances does not significantly change the results of the quantitative risk analysis.[6]

Comparison with Results from Kinetic Theory

In this study collision risk was determined via direct statistical analysis of miss distances that were generated by simulation of long-term orbital evolution. Another method frequently used in orbital debris and collision risk analysis is the application of kinetic theory. Similar to the theory of molecular gases, this method considers the threat objects to be randomly and independently distributed within a given volume of space. The average physical number density distribution of objects in the volume is used to represent a scaled version of the spatial probability density function throughout the volume. In addition to assuming a random distribution in position, the velocity vectors of the threat objects are assumed to be randomly and independently distributed, thereby maintaining the randomness of the position distribution during the time period that the primary object occupies the threat object field. With these assumptions collision events in nonoverlapping time intervals remain independent, and the probability of collision can be modeled via a Poisson process.[7] For collision probabilities that are much smaller than unity, the following simple formulation can be used:

$$p_c = \int_{I_{\mathrm{tf}}} \rho v A_{cc}\,\mathrm{d}t \tag{3}$$

I_{tf} is the cumulative time interval during which the primary object occupies the threat field. The relative velocity v between primary object and threat object is averaged over all threat objects in the local field. The integration is typically performed by discretizing the threat object field by altitude and latitude bins and then summing over the bins that are penetrated by the primary object. To the extent of the authors' knowledge, kinetic theory is the baseline formulation of most, if not all, currently existing debris environment evolution models, including EVOLVE[8] and IDES,[9] mainly as a result of its simplicity of implementation.

The application of kinetic theory to orbital collision problems is reasonably accurate as long as the motion of the threat objects resembles an ensemble of randomly moving objects within the volume and time of interest. This situation is generally the case for many LEO analyses in which the long-term collision risk posed to a primary object by a large group of background objects is to be assessed. In LEO various orbital processes have the effect of randomizing the threat object positions over long time periods. Nodal regression and apsidal rotation cause individual background object orbits to trace out a torus about the Earth.[10] Because nodal and apsidal regression rates are dependent upon orbital semimajor axis, eccentricity, and inclination, the variety of background objects will regress at different rates, thereby dismantling any short-term correlation between orbit orientations. In addition, variability in ballistic coefficients causes the orbital mean motions of different objects to increase over time at different rates, thereby randomizing relative in-track phasing.

In the case of the collision threat posed to GPS operational satellites by disposed vehicles, the most significant randomizing feature of the problem over the time period of interest (that is, 180 years) is the difference in mean motion. At a value of 1.5 deg per year, the relative nodal regression rate between the orbits of the disposed vehicle and operational orbits will produce a relative precession of only 270 deg over 180 years and hence is too slow to fully randomize the orbit orientations. Orbital mean motion does not change as a result of the absence of energy-dissipating forces at GPS altitudes, such as atmospheric drag. On the other hand, the operational satellites do among themselves have uniformly spaced ascending nodes, albeit deterministically and not randomly. Also, the lines of apsides are distributed in a relatively uniform manner throughout the orbit planes. As a result, it is unclear in advance how accurately the kinetic theory approach would determine the collision risk. For this reason direct statistical analysis was used in this study as the baseline approach.

As a point of comparison, the kinetic theory method was also applied as follows. The collision risk was computed from the perspective of a disposed vehicle as it flies through the GPS constellation shell. The altitude histogram used to derive the density field of the operational constellation is shown in Fig. 11. This altitude histogram was derived from the 200-year propagation of the constellation by applying the equations of Dennis for altitude distributions of individual satellites.[11] The average relative velocity v between disposed and operational object was estimated to be 4.86 km/s. The average collision cross-sectional area was computed from the collision radius d as $A_{\mathrm{cc}} = \pi d^2$. Equation (3) was evaluated by a computer program, which used the same MEANPROP orbital element history for the disposed vehicle that was used by the statistical analysis to model the penetration of the constellation. The program computed the time spent by the disposed vehicle in each constellation density field bin in order to compute the summed terms in the discrete representation of the integral. The 200-year collision probability for a single disposed vehicle vs the constellation was computed. Disposal in each plane was considered. The resulting collision probabilities for each plane, compared with that computed from direct statistical analysis, are shown in Fig. 12. In this case the results for a keep-out radius of 4.6 m are shown, that is, for collisions that can result in either damage or significant amounts of debris for a block IIF design. In addition, constellation orbit eccentricity growth and rectification was modeled in the miss distance simulation used for this comparison.[6] From this, it is seen that the kinetic theory method predicts a long-term average collision probability growth rate, which is higher than the average growth rate obtained via direct statistical analysis by a factor varying from 1.1 to 1.9.

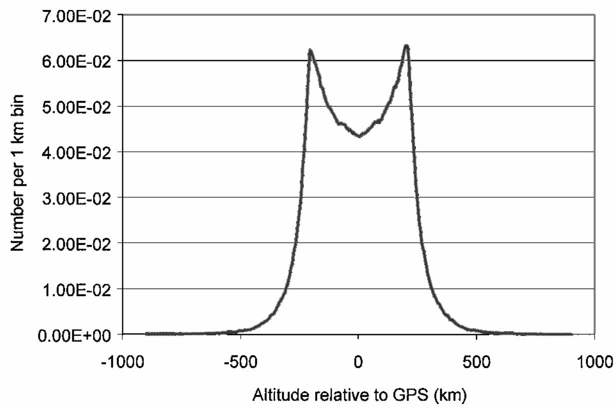

Fig. 11 Constellation altitude histogram used to compute spatial density field for kinetic theory analysis.

Effect of Disposal Orbit Population Growth

To assess the complete collision risk posed by disposal orbit instability, it is necessary to account for the growth rate of the disposal orbit population. For this analysis a long-term average replacement rate of 2.24 GPS satellites per year was used. Accounting for both decommissioned GPS vehicles and EELV upper stages, the long-term disposal orbit population rate is then 4.48 per year.

The preferred approach would be to directly simulate the population growth process so that the effect on miss distances of the variation in relative orbit geometry (primarily argument of perigee) between operational vehicle and disposed vehicle at epoch of disposal orbit insertion is obtained. This approach would be very demanding computationally, even in a parallel processing environment. In this study an approximate method that effectively exploits the fact that the collision probability for a single disposed vehicle was evaluated for the average eccentricity growth case was employed. The eccentricities and arguments of perigee of the operational vehicles in the prorated MCS data set were fairly randomly distributed among the orbit planes, and focusing on the average eccentricity growth scenario yields a good representation of the average geometry. Hence, although the resulting probability vs time profile might not be the precise average over its ensemble space, it is assumed to be a good approximation.

The probability vs time profile for each newly disposed vehicle is produced by time shifting the average probability vs time profile by the difference between the simulation case epoch and the disposal epoch. This generates a series of time-shifted profiles as shown in Fig. 13. At each time point the total collision risk is then obtained by summing over all of the nonzero valued profiles at that time. It is assumed that this method of summing of averages will yield a good approximation to the average of the sum of many nonaverage profiles that might occur in reality. The resulting overall collision probability over time for the Block IIF design is shown in Fig. 14.

Although probability of occurrence of damaging and debris-creating collisions is a desired metric of risk, the keep-out radii that are used to compute them are strongly dependent on vehicle design. Over time intervals of decades, the vehicle design will almost certainly change. To overcome this obstacle in obtaining a meaningful estimate of risk to the constellation, it is useful to consider the probability distribution of minimum miss distance over time intervals of interest. Figure 15 shows close approach distances and their likelihood of occurrence for several time intervals. The figure shows that close approaches at distances less than a kilometer begin to occur with significant certainty between 20 and 60 years. Although vehicle dimensions and corresponding collision distances will most likely remain within a few tens of meters, miss distances on the order of several hundred meters could have an operational impact. Currently operational tracking systems would yield position knowledge errors for the disposed vehicles on the order of a few kilometers, and they would not be able to resolve distances of hundreds of meters. As a result, it might be necessary to either improve tracking accuracy or

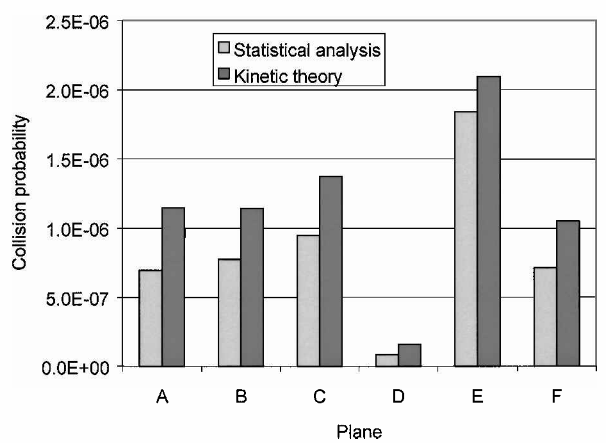

Fig. 12 Comparison of collision probabilities computed by the kinetic theory method and direct statistical analysis.

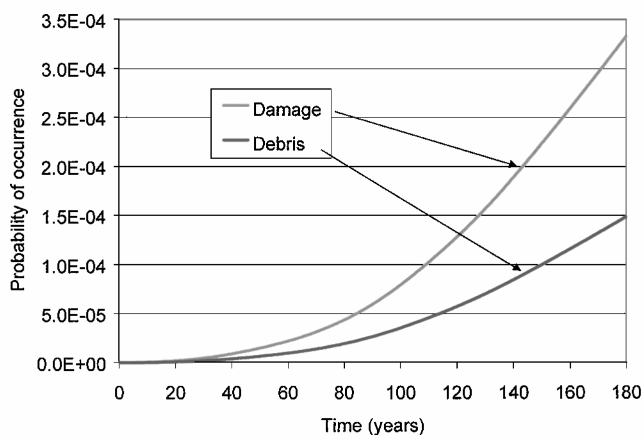

Fig. 14 Collision risk posed to a 28-satellite GPS constellation by the 500-km-perigee disposal orbit population (EELV upper stages and GPS decommissioned satellites).

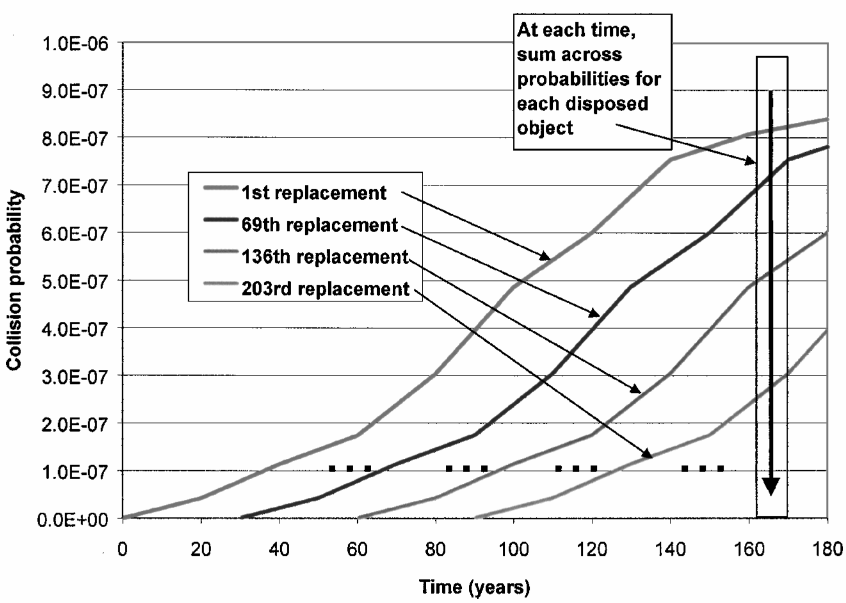

Fig. 13 Generation of the total collision risk caused by disposal orbit population growth by summing time-shifted average probability of collision time profiles.

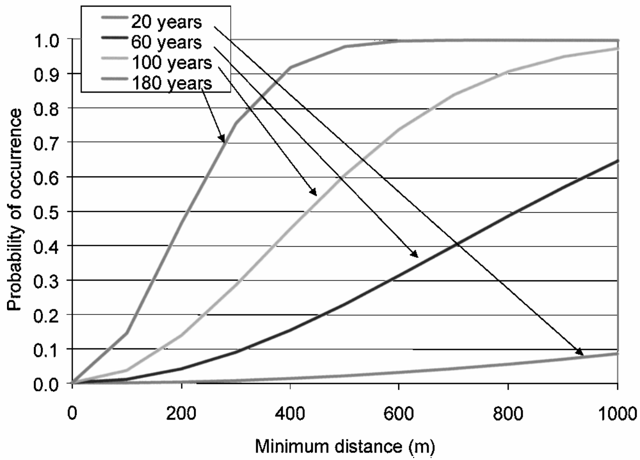

Fig. 15 Close approach distances that will occur over various time intervals and associated likelihood of occurrence.

plan for close approach warnings and maneuvers to ensure clearance that is larger than the position knowledge error. A complete assessment of the collision risk should include not only the probability of collision, but also any operational and tracking system impacts.

Conclusions

A preliminary analysis of the collision risk to the operational GPS constellation associated with disposal orbit instability was performed. Orbit eccentricity growth over several decades results in eventual penetration of the GPS operational altitude range by disposed vehicles. Selecting disposal orbits with increasingly higher perigees effectively defers initial constellation penetration to future decades, but the subsequent penetration of the remainder of the constellation occurs more rapidly.

The average collision risk associated with the 500-km disposal orbit, which is recommended by the U.S. Government Debris Mitigation Guidelines, was determined for the next 180 years. A direct statistical analysis was used to properly address the random and deterministic aspects of long-term orbital evolution. In addition, the widely used kinetic theory method was applied in order to obtain an understanding of its relative accuracy when applied to the GPS orbital regime. This preliminary analysis indicates that the average collision probability posed to the operational constellation assuming the Block IIF design will be low for the next two centuries. However, close approach distances on the order of hundreds of meters will start occurring between 20 and 60 years. This might impact constellation operations, depending on the tracking technology that will be available for the disposed vehicles.

Issues not considered in this study include the sensitivity of collision risk to nonaverage eccentricity growth scenarios; operational impacts, such as collision avoidance maneuver frequency; and the threat posed to the operational constellation by debris resulting from the intracollision risk between disposed vehicles. Should any of these issues prove to be a driver of requirements, a recommended way to control disposal orbit eccentricity growth is to target the initial eccentricity and argument of perigee within a predetermined window. The details of this mitigation approach are discussed in Ref. 1.

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References

[1]Gick, R. A., and Chao, C. C., "GPS Disposal Orbit Stability and Sensitivity Study," American Astronautical Society, AAS Paper 01-244, Feb. 2001.

[2]Chao, C. C., "MEO Disposal Orbit Stability and Direct Reentry Strategy," American Astronautical Society, AAS Paper 00-152, Jan. 2000.

[3]McClain, W. D., "A Recursively Formulated First-Order Semianalytic Artificial Satellite Theory Based on the Generalized Method of Averaging," Computer Sciences Corp., CSC/TR-77/6010, Greenbelt, MD, Nov. 1977.

[4]Jenkin, A. B., "Probability of Collision During the Early Evolution of Debris Clouds," *Acta Astronautica*, Vol. 38, Nos. 4–8, 1996, pp. 525–538.

[5]Chobotov, V. A., and Johnson, C. G., "Effects of Bunching on the Probability of Collision in Geosynchronous Orbit," *Journal of Spacecraft and Rockets*, Vol. 31, No. 5, 1994, pp. 895–899.

[6]Jenkin, A. B., and Gick, R. A., "Collision Risk Associated with Instability of MEO Disposal Orbits," *Proceedings of the Third European Conference on Space Debris*, Vol. 1, ESA, Noordjwijk, The Netherlands, 2001, pp. 471–476.

[7]Stark, H., and Woods, J. W., *Probability, Random Processes, and Estimation Theory for Engineers*, 1st ed., Prentice–Hall, Upper Saddle River, NJ, 1986, pp. 28–31.

[8]Ojakangas, G. W., Anz-Meador, P. D., and Reynolds, R. C., "Orbital Debris Environment," AIAA Paper 90-3863, Sept. 1990.

[9]Walker, R., Hauptmann, S., Crowther, R., Stokes, H., and Cant, A., "Introducing IDES: Characterizing the Orbital Debris Environment in the Past, Present, and Future," American Astronautical Society, AAS Paper 96-113, Feb. 1996.

[10]Chobotov, V. A. (ed.), *Orbital Mechanics*, 2nd ed., AIAA Education Series, AIAA, Reston, VA, 1996, pp. 307–310.

[11]Dennis, N. G., "Probabilistic Theory and Statistical Distribution of Earth Satellites," *Journal of the British Interplanetary Society*, Vol. 25, 1972, pp. 333–376.

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